Modelling and Analysis of Fractional Capacitors

Madhab Chandra Tripathy, Sribatsa Behera

Abstract— **The aim of the paper is to explore both theoretically and experimentally the modelling of different in-house developed PMMA coated fractional capacitors [5]. The design realization and behaviour of Polymethyl methacrylate (PMMA) coated probe has been studied in different polarizable medium like Ph 9.2, Ph 4.0, tape water and distilled water. Different frequency responses are obtained and a corresponding mathematical model has been proposed from experimental data. The parameters of modelled data have been found using least square estimation technique.**

Index Terms— **Fractional capacitor, Constant phase element, Polymethyl methacrylate (PMMA)**

I. INTRODUCTION

The last two decades have seen considerable progress in research on fractional order device and system. Fractional order system which is based on fractional calculus, was mostly restricted as a research topic among the mathematicians only. But several applications of fractional calculus came in to light recently in electrochemistry [1], acoustics [2], thermal processes [3], diffusion-wave [4], and analog circuits [5, 6, 7, 8]. The behaviors of these systems were found to be better explained by fractional-order differential equations rather than using classical integer order differential equations [9.10].

A passive circuit element that gives phase angle between 0 to -90 degree and remains constant with frequency is called a fractional capacitor (FC). The impedance of a fractional capacitor (FC) is expressed $Z = 1/Cs^{\alpha}$, where C is the fractional capacitance and α (0 < α ≤ 1) is its order. This fractional operator α can be used to physically interpret the voltage-current relationship of a fractional capacitor. In such element the phase difference between the voltage across its two terminals and current entering these terminals is $-\pi\alpha/2$. The unit of fractional capacitance is expressed as $F/s^{1-\alpha}$ where, `s' denotes second and F denotes farad. Similarly, the magnitude characteristic of an ideal FC is a straight line with slope -20 dB/decade while the phase angle remains constant for all frequencies.

Conventional capacitors and inductors are characterized by first order differential equations. In recent time, the fabrication of fractional capacitor (also called fractance, constant phase element etc.) that obeys fractional calculus and can be used as two terminal passive device, has been reported [5]. This brings in to light the possibility of several circuit applications of fractional order circuits. However, most of the studies so far carried out are restricted to simulation only . In some cases, experimentations are carried out by using RC-ladder network [7] as a fractional capacitor, which is an approximate technique. The objective of the present paper is to explore both theoretically and experimentally the modeling of fractional order capacitors using in-house developed PMMA-coated fractional capacitors [5].

Here, the electrical interaction of metal-insulator-liquid interface of FC has been studied and an equivalent circuit model has been proposed by considering the double layer capacitance at interface. The proposed model is a two constant phase element (CPE) model where the FC shows two constant phase angle region while charactering with a precision LCR meter in Z and θ mode in frequency range 100 Hz to 1 MHz

This paper is organized as follows: Sect.2 presents the realization of fractional order capacitors. Section3 presents the modeling the interface layer behavior of FC. The results and discussions of the proposed model have been presented in Sect.4, and concluding remarks are summarized in Sect.5.

II. REALIZATION OF FRACTIONAL CAPACITORS

PMMA-coated FC is a linear two-terminal device similar to resistor, capacitor and inductor. It is realized by dipping a capacitive type probe in a polarizable media (Fig. 1). The capacitive type probe consists of double sided metal plated insulator, coated with porous film of Polymethyl metha crylate (PMMA) polymer [5] dipped in a ionic solution. Generally, two types of metal electrodes are used for fabrication of FC i.e., copper electrode [5] and platinized silicon electrode [36].

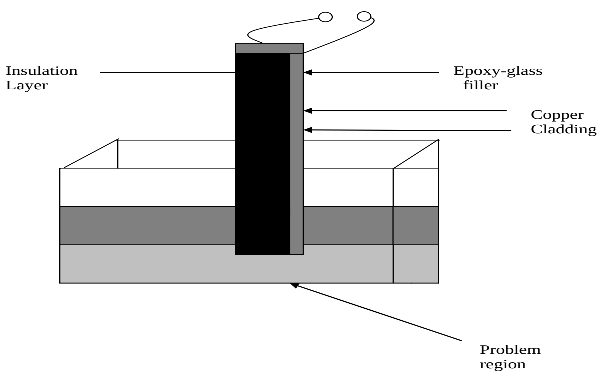

Fig. 1: Cross-sectional view of a FC

The fractional behavior of fabricated FC arises due to anomalous diffusion of ions through the porous film of PMMA deposited on the copper electrode. This porous nature of the electrode surface is observed under Scanning Electron Microscope (SEM) (JEOL, JSM-5800 Scanning Mi-croscope) as shown in Fig. 2. The SEM image gives an idea about the structure and dimensions of pores on electrode

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surface. It can be observed that pores on the surface of electrodes are more or less circular in nature and their diameters are in the order of micrometer. The constant phase angle (CP A) behavior of FCs is exhibited due to porosity of the electrode surface.

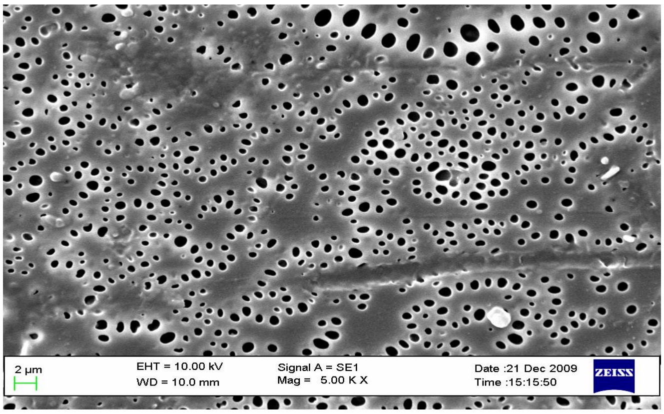


Fig. 2: SEM picture of the surface of PMMA coating FC

In literature, it has been reported that the fractional-order ` 'α of fabricated FC depends on the following parameters
(i) Thickness of porous PMMA coating
(ii) Conductivity of the ionic solution
(iii) Area of contact of the electrode surface with the polarizable medium.
This knowledge of dependence of α `on the above mentioned parameters is utilized to realize different values of α.
Most recently, the authors in [11] have developed pack aged form of FC as shown in Fig. 1.5 which can be conveniently used in practical analog circuits. This pack aged
FC increases its portability and easier use in practical circuits.

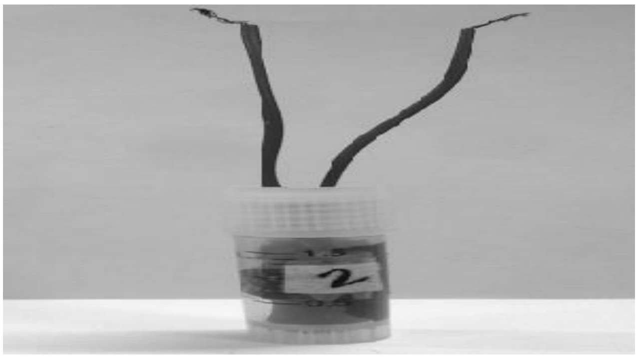
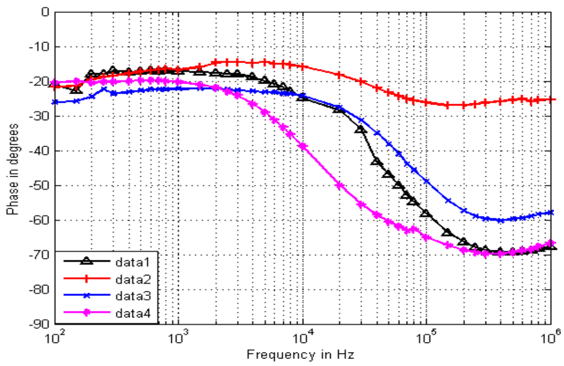


Fig. 3 Photograph of pack aged FC

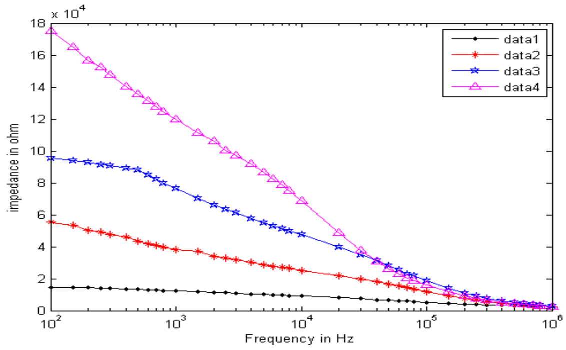
The impedance of porous electrode depends on the pore shape. At high frequency the impedance does not depend on faradic reactions and hence is said to be ideally polarizable. The real electrode surface consists of many pores of different dimensions as shown from SEM figure, it is difficult to model the porous electrode using a single pore model as in transmission line model as described in [12] by J. Bisquert et.el. However, recently the complex pore structure was modeled [13], considering all distributed pores as cylindrical shape.

III. Modeling of Interface Layer of Fractional Capacitors (FCs)

The measurement of impedance and phase angle of the FOEs is done with a precision LCR meter in Z and θ mode i.e. the equivalent impedance in polar form. An ac excitation of variable frequency and amplitude of 1V peak to peak is applied to the electrode. The Z and θ of the probe partially dipped in tap water, pH 9.2 solutions, and distilled water, were measured separately at different coating thickness for range of frequency 100 Hz to 1 MHz as shown in figure.



(a) Phase plot



(b) Magnitude plot

Fig. 4 Behavior of constant phase angle characteristics of FCs Data 1: Coating thickness=20 µm in tap water, Data 2: Coating thickness=10 µm in pH 9.2 solution, Data 3: Coating thickness= 5 µm in distilled water, Data 4: Coating thickness= 15 µm in tap water.

From phase plot, it is clear that the all probes show constant phase angle behavior in two frequency range, i.e. at low frequency range and at very high frequency range. This constant phase angle behavior is due to surface roughness and heterogeneities, porosity, variation of coating composition, non-uniform of current and potential distribution, and slow adsorption reactions. Due to the above characteristics, the interface layer behavior of FCs shows the 'distribution of time constants property' in the entire frequency spectrum. It is to be noted that at the high frequency range, instantaneous polarization occur due to displacement of electron and so the ions get less time to diffuse electrode to electrolyte or vice-versa. This results a low time constant in high frequency region and termed as τ_2 . Similarly in low frequency region, static polarization occurs due to orientation of dipole and hence ions get considerable more time to diffuse through

interface layer . Also this causes a higher relaxation time constant τ_1 at low frequency region is larger as compared to the high frequency region..

Due to instantaneous polarization at high frequency region and static polarization at low frequency region, the interface layer characteristics of FC may be considered as two Constant phase element (CPE) model. Also the existence of two CPE model can be seen from Nyquist diagram of FCs (Fig. 5) based on experimentally obtained impedance (Z) and phase angle (θ) of above four FOEs as shown in fig. 12.

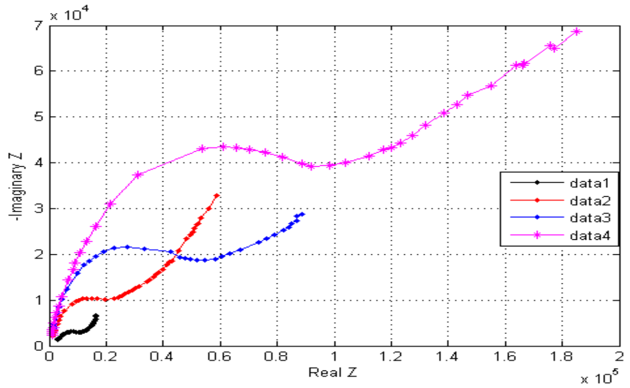


Fig. 5: Niquist Diagram

Hence the proposed equivalent circuit of FCs may be considered as two CPE connected in parallel as shown in Fig. 6.

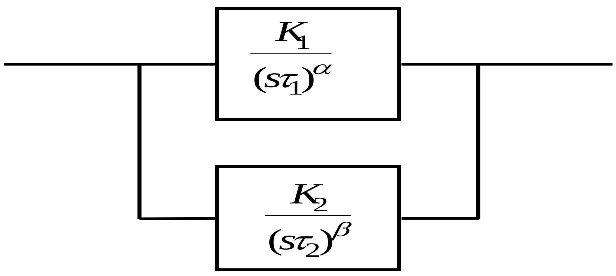


Fig. 6 Two CPE models of FCs

Where α β are exponent factor which depends on porosity, coating thickness surface roughness etc

$k_1 k_2$ are constants depends on capacitive dispersion

τ_1 τ_2 are relaxation time constants depends on static and instantaneous polarization

K is a resistive component which is generally the leakage resistance/ solution resistance of the dipping solution

IV. Results and Discussion

The overall equivalent impedance of the proposed model can be written in phase and magnitude form, which are as follows

$$Z = \frac{k_1\, k_2}{\sqrt{a^2 + b^2 + 2ab\cos\pi(\beta - \alpha)/2}}$$

$$\tan\theta = -\frac{a\sin\pi\beta/2 + b\sin\pi\alpha/2}{a\cos\pi\beta/2 + b\cos\pi\alpha/2}$$

Where $a = k_1(w\tau_2)^{\beta}$ and $b = k_2(w\tau_2)^{\alpha}$

The iterative form of equations of phase and magnitude can be as follows

$$f = z^2\Big(k_1^{\,2}(w\tau_2)^{2\beta} + k_2^{\,2}(w\tau_2)^{2\alpha} + 2k_1k_2(w\tau_1)^{\alpha}(w\tau_2)^{\beta}\cos\pi(\beta-\alpha)/2\Big) - k_1^{\,2}k_2^{\,2} \leftarrow magnitude$$

$$g = \tan\theta(k_1(w\tau_2)^{\beta}\cos\pi\beta/2 + k_2(w\tau_1)^{\alpha}\cos\pi\alpha/2) + (k_1(w\tau_2)^{\beta}\sin\pi\beta/2 + k_2(w\tau_1)^{\alpha}\sin\pi\alpha/2 \leftarrow phase$$

To estimate k_1, k2, τ1, τ2, β, α the above non linear equation are solved iteratively using Newton Rapson algorithm. The value of various parameters (k_1, k2, τ1, τ2, β, α) has been found in different conditions and compared with the experimental plots. Here, three nos of FCs are taken to validate the model.

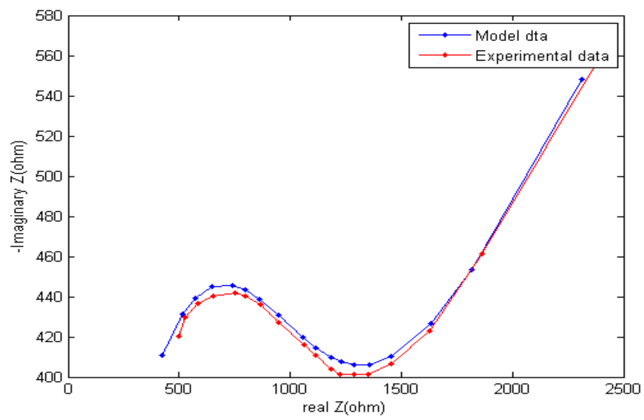


Fig. 7(a) Niquist Diagram (FC1)

V. Conclusion

In this paper, the design, realization, analysis and modeling of fractional capacitors has been presented. The behavior of PMMA coated FC has been studied in different polarizable medium like Ph 9.2, Ph 4.0, tape water and distilled water.

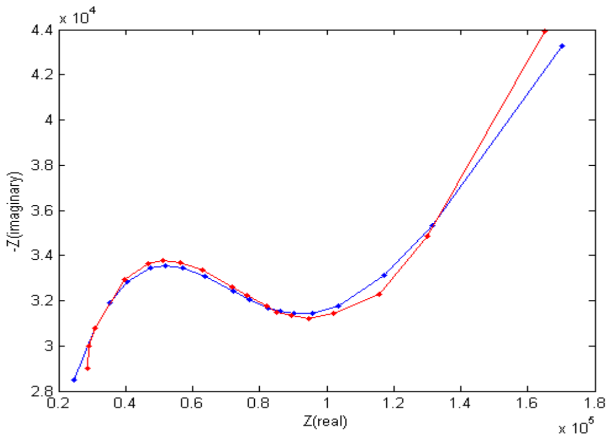


Fig. 7(b) Niquist Diagram (FC2)

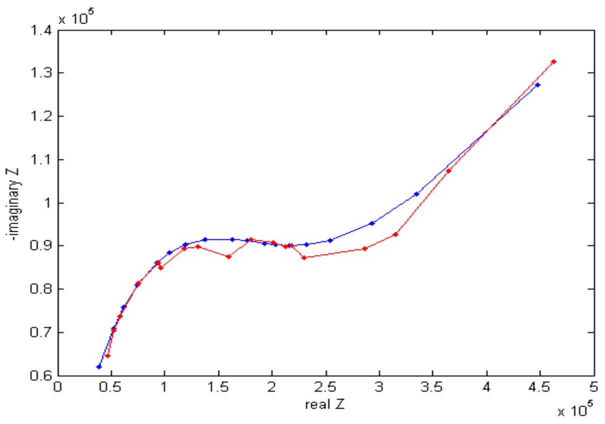


Fig. 7(c) Niquist Diagram (FC3)

Fig. 7 Comparison of model data with Experimental data of three different FCs

It is conclude that the PMMA coated FCs have a porous surface as seen from SEM picture. The pore surface has distributed pore size (different dimension) for which the FCs are assumed to have distributed time constant. These distributed time constant contribute constant phase angle behaviors. Several experiments have been conducted to determine Z and θ of FCs using precision LCR meter, which shows that phase angle θ of the realized FCs are approximately constant at low frequency and high frequency range.

Also, the mathematical models of FCs have been proposed from experimental data and the parameter of modeled data has been found using Newton-Rapson method. With this parameter the niquist diagram of the different FCs are plotted and compared with experimental data. It is to be noted that FCs are nowadays plays significant role in fractional order filter []14,15, 16]

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